

Hybrid Transform Based Denoising with Block Thresholding

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Abstract:

A frequently used approach for denoising is the shrinkage of coefficients of the noisy signal representation in a transform domain. This paper proposes an algorithm based on hybrid transform (stationary wavelet transform proceeding by slantlet transform); The slantlet transform is applied to the approximation subband of the stationary wavelet transform. BlockShrink thresholding technique is applied to the hybrid transform coefficients. This technique can decide the optimal block size and thresholding for every wavelet subband by risk estimate (SURE). The proposed algorithm was executed by using MATLAB R2010a minimizing Stein's unbiased with natural images contaminated by white Gaussian noise. Numerical results show that our algorithm competes favorably with SWT, and SLT based algorithms, and obtain up to 1.23 dB PSNR improvement.

Keywords-Stationary wavelet transforms (SWT); Slantlet transform (SLT); block threshold; image denoising.

INTRODUCTION

Noise removal is one of the most common and important preprocessing steps in image processing. The wavelet transform has been a powerful and widely used tool in image denoising because of its high energy compaction and multiresolution properties. Donoho and Johnstone proposed an innovative nonlinear denoising scheme *VisuShrink* [1] which thresholds the wavelet detail coefficients for one dimension (1-D) signals. *VisuShrink* is simple and efficient. Its denoising procedures can be stated as follows. First perform a wavelet transform on the observed data which are corrupted by additive white Gaussian noise (AWGN), then apply soft or hard thresholding to the wavelet detail coefficients using the universal threshold, last obtain the denoised signal by performing the inverse wavelet transform to the thresholded wavelet coefficients. Donoho and Johnstone also proposed the data-driven adaptive *SureShrink* [2] method in order to remedy the *VisuShrink*'s drawback. *SureShrink* denoised images include more significant wavelet coefficients thus alleviating the blurring problem produced by *VisuShrink*, generating more detailed images. *BayesShrink* [3] proposed by Chang et al. is also a data-driven adaptive image denoising method. Its denoising results are similar with the *SureShrink*'s. *VisuShrink*, *SureShrink* or *BayesShrink* denoising techniques are all to threshold the wavelet detail coefficients term by term based on their individual magnitudes.[4] explores the properties of the previous thresholding techniques in wavelet denoising in addition to Feature Adaptive Shrinkage.

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Cai [5] used the block thresholding scheme according to the blockwise James-Stein rules (*BlockJS*). The block thresholding simultaneously keeps or kills all the coefficients in groups rather than individually, enjoys a number of advantages over the conventional term-by-term thresholding. The block thresholding increases the estimation precision by utilizing the information about the neighbor wavelet coefficients. Cai and Silverman [6] considered the overlapping block thresholding estimators. Chen et al. [7] applied Cai and Silverman's estimation method to image denoising, called *NeighShrink*. *NeighShrink* outperforms *SureShrink*. Unfortunately, the block size and threshold level play important roles in the performance of a block thresholding estimator. The local block thresholding methods mentioned above all have the fixed block size and threshold and same thresholding rule is applied to all resolution levels regardless of the distribution of the wavelet coefficients. Recently, Cai and Zhou [8] proposed *SureBlock* estimation method. For 1-D data, the *SureBlock* estimator is a data-driven approach to empirically select both the block size and threshold at individual resolution levels. It performs better than *SureShrink* and *BlockJS* and is also easy to implement. Dengwen and Xiaoliu [9] extend Cai and Zhou's *SureBlock* estimator to image denoising. They proposed a block thresholding scheme and call it *BlockShrink*. There are a number of subbands produced from wavelet decomposition on an image. Every subband is needed to divide into a lot of square blocks. *BlockShrink* can select the optimal block size and threshold for the given details subband by minimizing Stein's unbiased risk estimate. Experimental results show that *BlockShrink* outperforms significantly the classic *SureShrink* by the term-by-term thresholding and *NeighShrink* with the fixed overlapping block size and threshold proposed by Chen et al. At other side, the classical discrete wavelet transform is not a time – invariant transform. The DWT of a translated version of a signal X is not, in general, the translated version of the DWT of X . Stationary wavelet transform was introduced in 1996 to make the wavelet decomposition time invariant [10]. This improves the power of wavelet in signal-denoising. In 1999, Selesnick [11] proposed a wavelet like filters known as slantlet filters which can provide better time localization and better signal compression compared to the conventional classical discrete wavelet transform (DWT). Panda and Meher [12] have proved these facts through simulation. In this paper we have explored the importance of stationary wavelet transform and slantlet transform in suppressing noise from colored digital images. The hybrid transform domain consists of stationary wavelet transform and slantlet transform applied to the approximation subband of the first one, block thresholding scheme which utilizes the information about the neighbor transform coefficients and can select the optimal block size and threshold for every subband is applied to all obtained subbands.

THE PROPOSED SWT-SLT TRANSFORM

In order to obtain the benefit of both SWT and SLT transforms in signal decorrelation, we applied the first one to the $N \times N$ (where $N=2^j, j=1,2,3,\dots$) noisy image, the output is $2N \times 2N$ signal which is divided into LL, LH, HL, and HH subbands. The slantlet transform is applied to the LL subband of the previous stage.

The Stationary Wavelet Transform

The SWT algorithm is very simple and is close to the DWT one. More precisely, for level 1, all the ϵ -decimated DWT (only two at this level) for a given signal can be obtained by convolving the signal with the appropriate filters as in the DWT case but without downsampling. Then the approximation and detail coefficients at level 1 are both of size N , which is the signal length [13]. This can be visualized in Fig. 1. The general step j convolves the approximation coefficients at level $j-1$, with upsampled version of the appropriate original filters, to produce the approximation and details coefficients at level j .

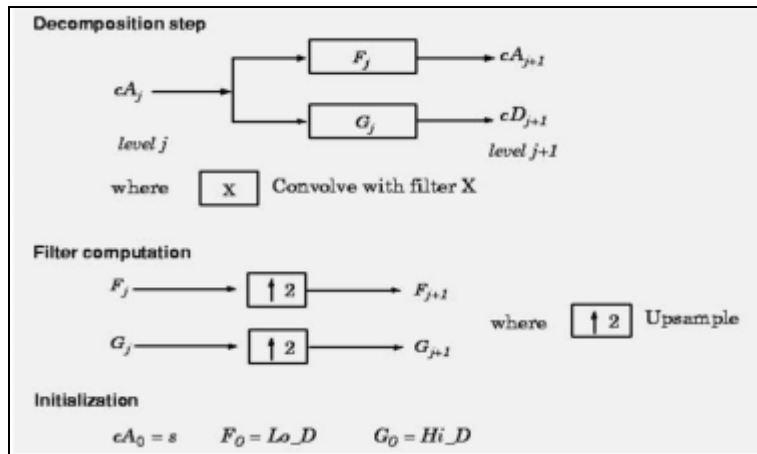


Figure 1. Decomposition step, one dimensional s wt.

The Slantlet Transform

Wavelet transform offers relatively efficient representation of piecewise smooth signals and is an effective tool for the application that needs multiresolution analysis. It provides short windows at high frequencies and long windows at low frequencies. One of its disadvantages is that for a fixed number of zero moments, it cannot yield a discrete-time basis that is optimal with respect to time localization. Selesnick [11] proposed Slantlet transforms (SLT) in 1999, which can provide better time localization. The filterbank defined by him for Slantlet transform employs a similar parallel structure like DWT providing exactly a scale dilation factor of 2. Slantlet filters are piecewise linear filters which can be applied with shorter and shorter supports maintaining the features of DWT. Like DWT, Slantlet filters are orthogonal, provides octave band characteristics and multiresolution decomposition.

They also have the same number of zero moments like DWT [14]. The Slantlet filters are based on the parallel structure of DWT filter bank as shown in **Fig.2**. The SLT filterbank is implemented in form of a parallel structure, employing different filters for each scale whereas DWT is usually implemented in form of an iterated filterbank, utilizing a tree structure. The filter coefficients used in the slantlet filter bank as derived in by Selesnick [11] are:

$$G_0(z) = \left(-\frac{\sqrt{10}}{20} - \frac{\sqrt{2}}{4}\right) + \left(\frac{3\sqrt{10}}{20} + \frac{\sqrt{2}}{4}\right)z^{-1} + \left(-\frac{3\sqrt{10}}{20} + \frac{\sqrt{2}}{4}\right)z^{-2} + \left(\frac{\sqrt{10}}{20} - \frac{\sqrt{2}}{4}\right)z^{-3} \quad (1)$$

$$G_1(z) = \left(\frac{7\sqrt{5}}{80} - \frac{2\sqrt{55}}{80}\right) + \left(-\frac{\sqrt{5}}{80} - \frac{\sqrt{55}}{80}\right)z^{-1} + \left(-\frac{9\sqrt{5}}{80} - \frac{\sqrt{55}}{80}\right)z^{-2} + \left(-\frac{17\sqrt{5}}{80} + \frac{3\sqrt{55}}{80}\right)z^{-3} \\ + \left(\frac{17\sqrt{5}}{80} + \frac{3\sqrt{55}}{80}\right)z^{-4} + \left(\frac{9\sqrt{5}}{80} + \frac{\sqrt{55}}{80}\right)z^{-5} + \left(\frac{\sqrt{5}}{80} - \frac{\sqrt{55}}{80}\right)z^{-6} + \left(-\frac{7\sqrt{5}}{80} - \frac{3\sqrt{55}}{80}\right)z^{-7} \quad (2)$$

$$G_2(z) = \left(\frac{1}{16} + \frac{\sqrt{11}}{16}\right) + \left(\frac{3}{16} + \frac{\sqrt{11}}{16}\right)z^{-1} + \left(\frac{5}{16} + \frac{\sqrt{11}}{16}\right)z^{-2} + \left(\frac{7}{16} + \frac{\sqrt{11}}{16}\right)z^{-3} \quad (3) \\ + \left(\frac{7}{16} - \frac{\sqrt{11}}{16}\right)z^{-4} + \left(\frac{5}{16} - \frac{\sqrt{11}}{16}\right)z^{-5} + \left(\frac{3}{16} - \frac{\sqrt{11}}{16}\right)z^{-6} + \left(\frac{1}{16} - \frac{\sqrt{11}}{16}\right)z^{-7}$$

$$G_3(z) = Z^{-3}G_2(1/Z) \quad (4)$$

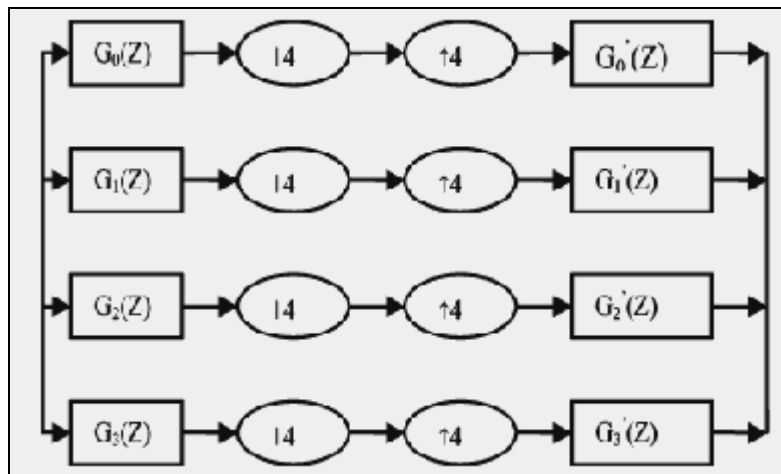


Figure 2. Two level equivalent filter bank structure.

In the SLT each filter has its length in power of 2 and the filterbank gives a reduction of $(2^i - 2)$ samples or supports for scale i and the reduction in support approaches one thirds as i increases (refer to [11], for details). In order to get better

coding performance by further decorrelates the coefficients of the input signal, two transforms can be used in cascading form [15]. In our proposed scheme, the stationary wavelet transform is followed by slantlet transform. **Fig. 3** shows the block diagram of this scheme

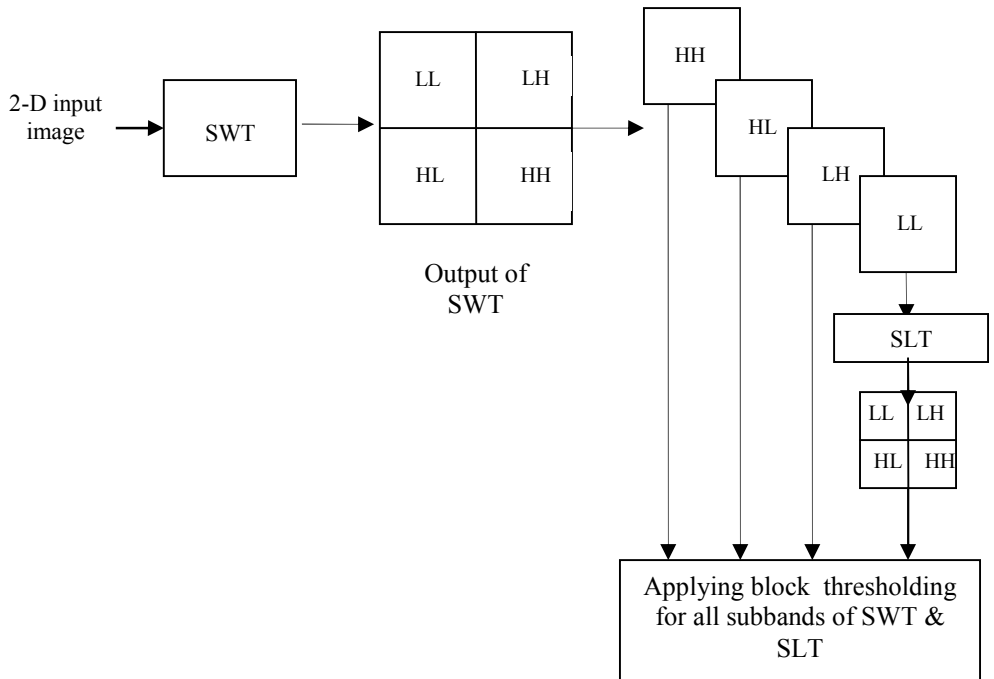


Figure 3. The proposed SWT-SLT transform

IMPLEMENTATION OF THE PROPOSED ALGORITHM

Cai and Zhou [8] proposed *SureBlock* thresholding method for 1-D signals described as follows. Suppose

$$w = \{w_i, i = 1, 2, \dots, d\} \text{ and}$$

$$S_b^2 = \sum_{i \in ib} w_i^2, ib = \{i : (b - 1)L + 1 \leq i \leq bL\}$$

on some block b of scale k where L is the block length. If S_b^2 is less than or equal to the threshold λ , then within the b -th block, we set the wavelet coefficient w_i to zero. Otherwise, we shrink it according to James-Stein rule

$$\hat{\theta}_i = w_i \left(1 - \frac{\lambda}{S_b^2}\right)_+ \quad \text{for } i \in ib$$

The optimal threshold λ and block size L is derived by minimizing Stein’s unbiased risk estimate (SURE).

$$(\lambda^s, L^s) = \arg \min_{\lambda, L} SURE(w, \lambda, L)$$

where

$$SURE(w, \lambda, L) = \sum_{b=1}^m SURE(w_b, \lambda, L) \text{ and}$$

$$SURE(w_b, \lambda, L) = L + \frac{\lambda^2 - 2\lambda(L - 2)}{S_b^2} I(S_b^2 > \lambda) + (S_b^2 - 2L) I(S_b^2 \leq \lambda)$$

$m=d/L$ is the number of blocks.

For simplicity d has been assumed to be divisible by L . Dengwen and Xiaoliu extended Cai and Zhou’s method to image denoising [9]. Suppose an image $\mathbf{X} \{X_{ij}\}$ is contaminated with Gaussian random noise with zero mean and variance σ_n^2 we observe

$$Y_{ij} = X_{ij} + \varepsilon_{ij} \quad i, j = 1, \dots, N \quad (5)$$

where N is some integer power of 2 and $\varepsilon = \{\varepsilon_{ij}\}$ is independent and identically Gaussian (normal) distributed (*iid*) $N(0, \sigma_n^2)$. The intention of image denoising is to construct an optimal estimate

$\hat{X} = \{\hat{X}_{ij}\}$ of $X = \{X_{ij}\}$ based on $Y = \{Y_{ij}\}$. They applied a two dimensional orthonormal wavelet transform to eq. (5) and there are a lot of subbands to be generated. For image denoising, a threshold technique is applied to all detail subbands LH, HL, and HH, while don’t change the approximation subband LL. For every specific detail subband, they use an $L_x \times L_y$ nonoverlapping to square block divide the subband. On the $b_1 \times b_2$ -th block, they compute

$$S_{b_1, b_2}^2 = \sum_{i \in ib_1} \sum_{j \in jb_2} w_{ij}^2 \quad (6)$$

where $ib_1 = \{i : (b_1 - 1)L + 1 \leq i \leq b_1 L\}$, $jb_2 = \{j : (b_2 - 1)L + 1 \leq j \leq b_2 L\}$.

The thresholding or shrinkage formula on the $b_1 \times b_2$ -th block, they compute

$$S_{b_1, b_2}^2 = \sum_{i \in ib_1} \sum_{j \in jb_2} w_{ij}^2 \quad (7)$$

where $ib_1 = \{i : (b_1 - 1)L + 1 \leq i \leq b_1L\}$, $jb_2 = \{j : (b_2 - 1)L + 1 \leq j \leq b_2L\}$.

The thresholding or shrinkage formula on the $b_1 \times b_2$ -th block

$$\hat{\theta}_{ij} = w_{ij} \left(1 - \frac{\lambda}{S_{b_1 b_2}^2}\right)_+ \text{ for } i \in ib_1 \text{ and } j \in jb_2 \quad (8)$$

Its SURE risk

$$\begin{aligned} SURE(w_{b_1 b_2}, \lambda, L^2) &= L + \frac{\lambda^2 - 2\lambda(L^2 - 2)}{S_{b_1 b_2}^2} I(S_{b_1 b_2}^2 > \lambda) \\ &+ (S_{b_1 b_2}^2 - 2L^2) I(S_{b_1 b_2}^2 \leq \lambda) \end{aligned} \quad (9)$$

The total risk

$$SURE(w, \lambda, L^2) = \sum_{b_1, b_2=1}^m SURE(w_{b_1 b_2}, \lambda, L^2) \quad (10)$$

where $m=N/L$. best threshold λ^s and block size L^s by minimizing $SURE(w, \lambda, L^2)$, namely

$$(\lambda^s, L^s) = \arg \min_{\lambda, L} SURE(w, \lambda, L^2) \quad (11)$$

The BlockShrink denoising procedure with the proposed hybrid transform can be summarized as follows:

For each band (Red, Green, and Blue) of the color image

1. A one-level 2-D orthogonal stationary wavelet transform SWT is performed.
2. Apply 2-D orthogonal slantlet transform to the LL subband of the SWT output.
3. Every subband is thresholded (i.e., LH, HL, and HH of the SWT output and the all subbands of the SLT). First we search the optimal threshold λ^s and block size L^s by (11). λ^s is one of all $\{S_{b_1, b_2}^2\}$ on the subband. We also limit the block size search range to be $1 \leq L \leq \left\lceil (N / 2^k)^{3/4} \right\rceil$. Then we obtain the estimate $\hat{\theta}$ of the noiseless hybrid transform coefficients θ by(8).
4. Inverse hybrid transforms the modified coefficients to obtain the denoised estimate image.

EXPERIMENTAL RESULTS

The proposed hybrid transform have been experimented with BlockShrink method using various noisy images and report the results for the six 256x256 color test images **Fig. 4**. They are contaminated with Gaussian random noise with different noise levels. **Fig. 5** shows the denoised image (Girls) for noise levels (0.007, 0.020) of the four algorithms. In practice, the noise standard deviation is unknown, to estimate it, a good estimator is the median of absolute deviation (MAD) using the HH subband coefficients of the SWT output [16]. Our results are measured by the PSNR in decibel (dB) defined as [17].

$$PSNR = 10 * \log_{10} \frac{255^2}{MSE} (dB) \quad (12)$$

$$\text{where } MSE = \frac{1}{N^2} \sum_{i,j=1}^N (X(i,j) - \hat{X}(i,j))^2$$

The overall PSNR of color image is

$$PSNR = 10 \log_{10} \frac{255^2}{(MSE(R) + MSE(G) + MSE(B)) / 3} \quad (13)$$

where X is the original image, \hat{X} is the estimate of X , and N^2 is the number of pixels. The denoised image is closer to the original when PSNR is higher. The proposed algorithm is compared with wavelet, slantlet, and stationary wavelet transform based algorithms.

Table 1 shows the PSNR performance of the four denoising algorithms. The best one of the denoised results is highlighted in bold font for each test set. From the results of **Table 1** we notice that the PSNR of the proposed algorithm are highest than other three methods.

The average increase of PSNR of the denoised image with respect to noisy one is approximately (6-7) dB, while the improvement with respect to other methods is up to 1.32 dB.



Figure (4) Test color images



Noisy image $\sigma = 0.003$
PSNR=30.17dB



Denoised using WT
PSNR=32.67dB



Denoised using SWT
PSNR=33.72dB



Denoised using SLT
PSNR=33.69dB



Denoised using Hybrid
PSNR=34.12dB

Figure(5) Girls test image for different denoising algorithms

Table 1 The PSNR of the proposed algorithm and WT, SWT, and SLT transform

Test images	Noise variance	Noisy Image PSNR dB	WT PSNR dB	SWT PSNR dB	SLT PSNR dB	Proposed PSNR dB
Balloon	0.003	30.78	34.11	34.70	34.68	35.15
	0.007	27.28	31.08	31.38	31.42	32.00
	0.009	26.26	30.13	30.49	30.39	31.06
	0.010	25.84	29.80	30.04	30.01	30.70
	0.020	23.04	27.22	27.36	27.31	28.10
	0.040	20.33	24.55	24.63	24.62	25.31
	0.060	18.79	23.00	23.06	23.07	23.90
Girls	0.003	30.17	32.67	33.72	33.69	34.12
	0.007	26.53	30.34	30.90	30.89	31.52
	0.009	25.48	29.57	30.03	30.01	30.82
	0.010	25.07	29.27	29.69	29.69	30.45
	0.020	22.22	26.98	27.24	27.34	28.27
	0.040	19.51	24.61	24.74	24.75	25.96
	0.060	18.06	23.19	23.34	23.43	24.72
Parrots	0.003	30.12	34.63	35.18	35.25	36.11
	0.007	26.48	31.55	31.80	31.87	32.95
	0.009	25.47	30.65	30.88	30.90	32.12
	0.010	25.02	30.25	30.47	30.44	31.65
	0.020	22.20	27.64	27.81	27.77	29.02
	0.040	19.52	25.02	25.15	25.10	26.47
	0.060	18.07	23.52	23.63	23.70	25.14
Peppers	0.003	30.24	35.14	35.50	35.46	36.28
	0.007	26.69	31.97	32.16	32.21	33.17
	0.009	25.65	30.99	31.13	31.09	32.34
	0.010	25.21	30.52	30.74	30.65	31.97
	0.020	22.46	27.78	28.07	27.89	29.24
	0.040	19.85	25.10	25.26	25.27	26.58
	0.060	18.37	23.62	23.74	23.74	25.08
Flower	0.003	30.08	35.08	35.58	35.51	36.44
	0.007	26.47	31.90	32.13	32.18	33.12
	0.009	25.42	30.87	31.15	31.22	32.35
	0.010	25.00	30.56	30.81	30.66	31.92
	0.020	22.18	27.81	28.03	28.01	29.24
	0.040	19.57	25.19	25.34	25.40	26.64
	0.060	18.09	23.65	23.85	23.89	25.28
Fence	0.003	30.03	34.25	34.94	35.02	35.89
	0.007	26.40	31.25	31.70	31.75	32.77
	0.009	25.36	30.38	30.67	30.78	31.90
	0.010	24.90	29.91	30.25	30.25	31.47
	0.020	22.02	27.36	27.46	27.52	28.81
	0.040	19.28	24.75	24.93	24.85	26.39
	0.060	17.76	23.25	23.51	23.50	25.10

Conclusion

In this paper, a new image denoising method is proposed. This method employs the stationary wavelet transform and slantlet transform as a hybrid scheme with BlockShrink as a block thresholding. The hybrid scheme is of applying the stationary wavelet transform to the noisy input image then applying the slantlet transform to the approximation subband of the previous stage in order to exploit the multiresolution analysis of both of them to achieve better decomposition for the input signal. The proposed method includes applying the block thresholding method to the all coefficients of the output transforms instead of details coefficients only, as followed in other denoising methods. This gives better results as shown in this paper that compared the proposed method with wavelet, stationary wavelet and slantlet transforms methods. The improvement of PSNR is by about 1.32 dB as compared with other single transforms.

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إزالة الضوضاء بالاعتماد على التحويلة الهجينة وتقنية عتبة الكتلة

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الخلاصة:

من أكثر طرق إزالة الضوضاء شيوعاً هو تقليص معاملات الإشارة المعرضة للضوضاء في مجال التحويلة. هذا البحث يقدم خوارزمية تعتمد على تحويلة هجينة تتألف من تحويلة الموجة المستقرة متبوعة بتحويلة المويّيل، حيث تطبق تحويلة المويّيل على معاملات التقريب لتحويلة الموجة المستقرة. تطبق تقنية العتبة (BlockShrink) على جميع معاملات التحويلة الهجينة الناتجة. تستطيع تقنية العتبة هذه إيجاد حجم الكتلة المثالي لكل جزء من تحويلة الموجة بواسطة تقليل (Stein's unbiased risk estimate). تم تطبيق الخوارزمية المقترحة باستعمال برنامج ماتلاب R2010a على صور طبيعية ملونة وملوثة بضوضاء من نوع (Gaussian) وقد أظهرت النتائج كفاءة أفضل مقارنة مع أنظمة إزالة الضوضاء المعتمدة على تحويلة الموجة، الموجة المستقرة، و تحويلة المويّيل. أعلى نسبة ربح بلغت (1.32dB) مقارنة بطرق إزالة الضوضاء الأخرى.