SOLUTION OF THE ELECTROMECHANICAL MACHINING PROBLEM USING THE COLLOCATION METHOD BASED ON BERNSTEIN POLYNOMIALS

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ABSTRACT

In this paper, we have solved the electrochemical machining problem (ECM) by using the collocation method based on Bernstein polynomials.

This paper is divided into three parts, in the first part we introduce a simple overview about the two-dimensional ECM problem (in polar coordinate system), while in the second part we consider the Bernstein polynomials (including its definition and properties). Finally, the third part consists of evaluating the approximate solution of the mathematical modeling related to the ECM problem.

1- INTRODUCTION

As an alternative to definition to mechanical machining problem, a piece of metal can sometimes be shaped by using it as an anode in electrolytic cell with an appropriately shaped cathode. This represent a moving boundary value problem, because the anode surface changing with time. The anode is moved towards the cathode, or vice versa, at a constant velocity and products of the erosion of the anode are swept a way by the electrolyte, which is pumped through the space between the electrodes, [3].

2- THE MATHEMATICAL FORMU-LATION OF THE PROBLEM [3]

The two-dimensional ECM problem will be considered for two electrodes (as shown in Fig.1). The space between the electrodes is filled by an appropriate electrolyte. A voltage is placed across the electrode and these causes are removal of material from the anode. We will solve the ECM problem of an anode surrounded by circular cathode, the conductivity in the gap between the electrodes is considered constant. This problem is formally identical to one-phase Stefan problem with zero heat capacity.

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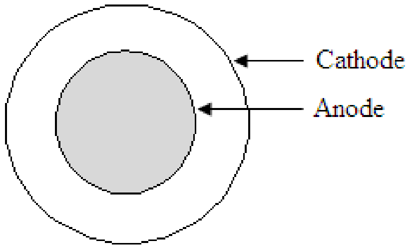

Fig.1: Basis configuration of ECM problem.

Referring to Fig.2, suppose that the anode is the shrinkage region A(t), with moving boundary Γ_t and Γ_0 denote the initial anode surface at t = 0. The region inside the cathode surface C is denoted by D and the region occupied by the electrolyte by D_t, so that D includes A(t) and D_t. Also, it is convenient to define the moving boundary Γ_t, $\forall\, t \geq 0$, by [3]:

$\Gamma_t = s(\theta, t)$ .... (1)

where:

$\Gamma = \{(r, \theta) : r = 1, 0 < \theta < 2\pi\}$

An approximation model for the process is given by:

$\tilde{\nabla}^2\tilde{\varphi} = 0$, in D .... (2)

$$\left.\begin{array}{l}\tilde{\varphi} = 0 \quad \text{on} \quad C \\ \tilde{\varphi} = g \quad \text{on} \quad \Gamma_t\end{array}\right\} \quad \text{..} \ (3)$$

$$\frac{d\tilde{R}}{dt} = M\tilde{\nabla}\tilde{\varphi}_a \quad \text{on} \quad \Gamma_t \quad \text{...................................} \ (4)$$

where D is defined as:

$D = \{(r, \theta, t), a \leq r \leq s(\theta,t), 0 \leq \theta \leq 2\pi, 0 \leq t \leq T\}$

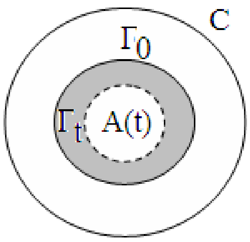

Fig.2: Moving boundary value problem Annular ECM problem.

It should be noted that the annular electrochemical machining boundary conditions (3) are based on the assumption that the effects of over potentials could be ignored.

The particular problem we treated in detail of circular cathode of radius c with anode inside it. We measure all lengths in units of αc, here α is a positive non-dimensional constant.

Then ∇ (the gradient operator of the non-dimensional length) and $\tilde{\nabla}$ are connected by $\nabla = \frac{\tilde{\nabla}}{\alpha c}$. We define further non-dimensional potential by $\tilde{\varphi} = g\varphi$, such that eqs.(2) and (3) becomes:

$\nabla^2\phi = 0$, in the electrolyte .... (5)

with boundary conditions:

$$\left.\begin{array}{lll} \phi = 0 & \text{if } r = a & \text{on the cathode} \\ \phi = 1 & \text{if } r = s(\theta, t) & \text{on the anode} \end{array}\right\} \quad \text{.............} \ (6)$$

A non-dimensional time variable T = (Mg αc)t is defined and eq.(4) becomes:

$$\frac{dR}{dT} = \nabla\phi\Big|_{anode} \quad \text{..} \ (7)$$

where:

$$L = \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

The characterization of potential equation is straightforward. However, the free boundary condition on the anode must be transformed into an expression in terms of the anode speed along each ray. Rewriting $\nabla\varphi$ and $\frac{dR}{dt}$ in polar coordinated system:

$$\left(\frac{\partial\phi}{\partial r}, \frac{1}{r}\frac{\partial\phi}{\partial\theta}\right) = \left(\frac{dr}{dt}, r\frac{d\theta}{dt}\right)$$

where r = s(θ, t) on the anode and this implies $\frac{\partial\phi}{\partial r} = \frac{dr}{dt}$ and $\frac{1}{r}\frac{\partial\phi}{\partial\theta} = r\frac{d\theta}{dt}$. Now:

$$\begin{aligned} \frac{dr}{dt} &= \frac{\partial s}{\partial r} + \frac{\partial s}{\partial\theta}\frac{d\theta}{dt} \\ &= \frac{\partial s}{\partial r} + \frac{\partial s}{\partial\theta}\frac{1}{r^2}\frac{\partial\phi}{\partial\theta} \\ &= \frac{\partial s}{\partial r} + \frac{\partial s}{\partial\theta}\frac{1}{s^2}\frac{\partial\phi}{\partial\theta} \quad \text{..................................} \ (8) \end{aligned}$$

It is convenient to replace $\frac{\partial\phi}{\partial\theta}$ by $\frac{\partial\phi}{\partial r}$. Since the tangential derivative vanishes on the anode, then:

$$\left(\frac{\partial\phi}{\partial r}\frac{\partial r}{\partial\theta} + \frac{\partial\phi}{\partial\theta}\frac{\partial\theta}{\partial\theta}\right)\Bigg|_{r=s(\theta,t)} = 0$$

Then:

$$\frac{\partial\phi}{\partial\theta} = -\frac{\partial s}{\partial\theta}\frac{\partial\phi}{\partial r} \quad \text{..} \ (9)$$

Now, substituting eq.(9) in eq.(8), so the gradient condition on the anode surface leads to:

$$\frac{\partial \phi}{\partial r} = \frac{\partial s}{\partial t} + \frac{\partial s}{\partial \theta}\frac{1}{s^2}\left[-\frac{\partial s}{\partial \theta}\frac{\partial \phi}{\partial r}\right]$$

Hence:

$$\frac{\partial s}{\partial t} = \frac{\partial \phi}{\partial r} + \frac{1}{s^2}\frac{\partial s}{\partial \theta}\frac{\partial s}{\partial \theta}\frac{\partial \phi}{\partial r}$$

$$= \frac{\partial \phi}{\partial r} + \left(\frac{1}{s}\frac{\partial s}{\partial \theta}\right)^2 \frac{\partial \phi}{\partial r}$$

Therefore:

$$\frac{\partial s}{\partial t} = \left[1+\left(\frac{1}{s}\frac{\partial s}{\partial \theta}\right)^2\right]\frac{\partial \phi}{\partial r}$$

or equivalently:

$$\left.\frac{\partial \phi}{\partial r}\right|_{r=s(\theta,t)} = \frac{\frac{\partial s}{\partial t}}{1+\left(\frac{1}{s}\frac{\partial s}{\partial \theta}\right)^2} \equiv f(\theta,t) \quad \text{.............} \ (10)$$

So, the final mathematical model of the problem is given by:

$\nabla^2\phi = 0$ in the electrolyte .... (11)

$\phi = 0$ on the cathode, where $r = a$ .... (12)

$\phi = 1$ on the anode, where $r=s(\theta,t)$ .... (13)

$\phi_r = f(\theta, t)$ on the anode............................. (14)

where:

$$f(\theta, t) = \frac{\frac{\partial s}{\partial t}}{1+\left(\frac{1}{s}\frac{\partial s}{\partial \theta}\right)^2}$$

3- BERNESTIEN POLYNOMIAL [1, 2]

Here, we would like to give the definition and some properties of Bernestien polynomials that are needed in this paper.

Definition (3.1):

The Bernstein polynomials of degree n are defined by:

$$B_{i,n}(t) = \binom{n}{i} t^i (1-t)^{n-i}$$

for i = 0, 1, …, n; where:

$$\binom{n}{i} = \frac{n!}{i!(n-i)!}$$

We usually set $B_{i,n} = 0$ if $i < 0$ or $i > n$. Also:

$$B_{i,n}(t) \geq 0, \quad \sum_{i=0}^{n} B_{i,n}(t) = 1$$

These polynomials are quite easy to write down the coefficients $\binom{n}{i}$ can be obtained from Pascal's triangle; the exponents on the t term increase by one as i increase; and the exponents on the (1 − t) decrease by one as i increase. In the simple cases, we obtain for $0 \leq t \leq 1$:

1- The Bernestien polynomials of degree 1 are:

$$B_{0,1}(t) = 1 - t$$

$$B_{1,1}(t) = t$$

2- The Bernestien polynomials of degree 2 are:

$$B_{0,2}(t) = (1-t)^2$$

$$B_{1,2}(t) = 2t(1-t)$$

$$B_{2,2}(t) = t^2$$

3- The Bernstein polynomials of degree 3 are:

$$B_{0,3}(t) = (1-t)^3$$

$$B_{1,3}(t) = 3t(1-t)^2$$

$$B_{2,3}(t) = 3t^2(1-t)$$

$$B_{3,3}(t) = t^3$$

Remark (3.2), [2]:

A relationship between Bernstein basis and functions power basis functions may be given next. Any Bernestien polynomial of degree n can be written in terms of the power basis, as follows:

$$B_{k,n}(t) = \sum_{i=k}^{n} (-1)^{i-k} \binom{n}{i} \binom{i}{k} t^i$$

Remark (3.3), [1]:

The derivative of Bernestien polynomials may be also given recursively, which may be derived using the definition of Bernestien polynomials, which is for $0 \le k \le n$:

$$\frac{d}{dt} B_{k,n}(t) = n(B_{k-1,n-1}(t) - B_{k,n-1}(t))$$

4- THE APPROXIMATE SOLUTION OF THE ECM PROBLEM USING THE COLLOCATION METHOD BASED ON BERNESTIEN POLYNOMIALS

The collocation method is one of the most common methods used to approximate the solution of the differential, integral equations, and partial differential equations. Here, we use the collocation method based on Bernestien polynomials to solve the ECM problem.

Now, in order to use the collocation method, we approximate the solution $\phi(r, \theta, t)$ as follows:

$$\phi(r, \theta, t) = \psi(r, \theta, t) + w(r, \theta, t)$$

where $\psi(r, \theta, t)$ is any function which satisfies the non-homogeneous boundary conditions and $w(r,\theta,t)$ any function satisfies the homogeneous boundary conditions.

One of the choices for $w(r, \theta, t)$ which fit our needs is modified here by using the Bernestien polynomials, as follows:

$$w(r, \theta, t) = (r - a)(r - s(r, \theta, t))^2 \sum_{i=0}^{2}\sum_{j=1}^{2} a_{ij} B_{i,n}(r) B_{j,n}(\theta)$$

where:

$$\phi_{ij}(r, \theta) = B_{i,n}(r)B_{j,n}(\theta), \ i = 0, 1, 2; \ j = 1, 2$$

which are chosen as Bernestien basis functions, hence:

$$w(r, \theta, t) = (r - a)(r - s(\theta, t))^2 [a_{01}(B_{0,1}(r) B_{0,1}(\theta)) + a_{02}(B_{0,2}(r)B_{0,2}(\theta)) + a_{11}(B_{1,1}(r)B_{1,1}(\theta)) + a_{12}(B_{1,2}(r) B_{1,2}(\theta)) + a_{22}(B_{2,2}(r)B_{2,2}(\theta))]$$

where a_{ij}, $i = 0, 1, 2$; $j = 1, 2$ are constants to be determined. For simplicity, rewrite this equation as follows:

$a_{01} = a_1$, $a_{02} = a_2$, $a_{11} = a_3$, $a_{12} = a_4$ and $a_{22} = a_5$

Therefore:

$$w(r, \theta, t) = (r - a)(r - s(\theta, t))^2 [a_1(B_{0,1}(r) B_{0,1}(\theta)) + a_2(B_{0,2}(r)B_{0,2}(\theta)) + a_3(B_{1,1}(r)B_{1,1}(\theta)) + a_4(B_{1,2}(r)B_{1,2}(\theta)) + a_5(B_{2,2}(r)B_{2,2}(\theta))]$$

In addition, for the non-homogeneous boundary conditions which satisfies $\phi = 1$ on the anode, and by using the mathematical inspection, we can take $\psi(r, \theta, t)$ to be as follows:

$$\psi(r,\theta,t) = \frac{r-a}{s(\theta,t)-a} + \frac{(r-a)(r-s(\theta,t))}{s(\theta,t)-a}\left(\frac{\dot{s}(\theta,t)}{1+\left(\frac{\overline{s}(\theta,t)}{s(\theta,t)}\right)^2} - \frac{1}{s(\theta,t)-a}\right)$$

Where $\dot{s}(\theta,t) = \frac{\partial s(\theta,t)}{\partial t}$ and $\overline{s}(\theta,t) = \frac{\partial s(\theta,t)}{\partial \theta}$, which is easily checked that when r = s(θ, t), then ψ(r, θ, t) = 1. Now, the Bernestien polynomials of degree 1 are:

$B_{0,1}(r) = 1 - r$, $B_{0,1}(\theta) = 1 - \theta$,

$B_{1,1}(r) = r$, $B_{1,1}(\theta) = \theta$

and the Bernestien polynomials of degree 2 are:

$B_{0,2}(r) = (1 - r)^2$, $B_{0,2}(\theta) = (1 - \theta)^2$,

$B_{1,2}(r) = 2r(1 - r)$, $B_{1,2}(\theta) = 2\theta(1 -\theta)$,

$B_{2,2}(r) = r^2$, $B_{2,2}(\theta) = \theta^2$

Therefore, inserting these polynomials in the definition of w(r, θ, t), we get:

$$\begin{aligned} w(r, \theta, t) &= (r - a)(r - s(\theta, t))^2[a_1(1 - r)(1 - \theta) + a_2(1 -r)^2(1 - \theta)^2 + a_3r\theta + a_4 (2r(1 - r)^2\theta(1 - \theta) + a_5r^2\theta^2] \\ &= (r - a)(r - s(\theta, t))^2[a_1(1 - r - \theta + r\theta) + a_2(1 -2r - 2\theta + r^2 + \theta^2 + 4r\theta - 2r^2\theta - 2r\theta^2 + r^2\theta^2) + a_3r\theta + a_4(4r\theta - 4r\theta^2 - 4r^2\theta + 4r2\theta^2) + a_5r^2\theta^2] \end{aligned}$$

Hence:

$$\frac{\partial w}{\partial r} = 2(r - a)(r - s(\theta, t))[a_1(1 - r - \theta + r\theta) + a_2(1 -2r - 2\theta + r^2 + \theta^2 + 4r\theta - 2r^2\theta - 2r\theta^2 + r^2\theta^2) + a_3r\theta + a_4(4r\theta - 4r\theta^2 - 4r^2\theta + 4r^2\theta^2) + a_5r^2\theta^2] + (r - s(\theta, t))^2 [a_1(1 - r - \theta + r\theta) + a_2(1 -2r - 2\theta + r^2 + \theta^2 + 4r\theta - 2r^2\theta - 2r\theta^2 + r^2\theta^2) + a_3r\theta + a_4(4r\theta - 4r\theta^2 - 4r^2\theta + 4r^2\theta^2) + a_5r^2\theta^2] + (r - a)(r - s(\theta, t))^2 [a_1(-1 + \theta) + a_2(-2 + 2r + 4\theta - 4r\theta - 2\theta^2 + 2r\theta^2) + a_3\theta + a_4(4\theta - 4\theta^2 - 8r\theta + 8r\theta^2) + 2a_5r\theta^2]$$

and also:

$$\frac{\partial^2 w}{\partial r^2} = 2(r - a)[a_1(1 - r - \theta + r\theta) + a_2(1 -2r - 2\theta + r^2 + \theta^2 + 4r\theta - 2r^2\theta - 2r\theta^2 + r^2\theta^2) + a_3r\theta + a_4(4r\theta - 4r\theta^2 - 4r^2\theta + 4r^2\theta^2) + a_5r^2\theta^2] + 2(r - s(\theta, t))[a_1(1 - r - \theta + r\theta) + a_2(1 -2r - 2\theta + r^2 + \theta^2 + 4r\theta - 2r^2\theta - 2r\theta^2 + r^2\theta^2) + a_3r\theta + a_4(4r\theta - 4r\theta^2 - 4r^2\theta + 4r^2\theta^2) + a_5r^2\theta^2] + 2(r - a)(r - s(\theta, t)) [a_1(-1 + \theta) + a_2(-2 + 2r - 4r\theta - 2\theta^2 + 4\theta + 2r\theta^2) + a_3\theta + a_4(4\theta - 4\theta^2 - 8r\theta + 8r\theta^2) + 2a_5r\theta^2] + 2(r - s(\theta, t))[a_1(1 - r - \theta + r\theta) + a_2(1 -2r - 2\theta + r^2 + \theta^2 + 4r\theta - 2r^2\theta - 2r\theta^2 + r^2\theta^2) + a_3r\theta + a_4(4r\theta - 4r\theta^2 - 4r^2\theta + 4r^2\theta^2) + a_5r^2\theta^2] + (r - s(\theta, t))^2[a_1(-1 + \theta) + a_2(-2 + 2r + 4\theta - 4r\theta - 2\theta^2 + 2r\theta^2) + a_3\theta + a_4(4\theta - 4\theta^2 - 8r\theta + 8r\theta^2) + 2a_5r\theta^2] + 2(r - a)(r - s(\theta, t)) [a_1(-1 + \theta) + a_2(-2 + 2r + 4\theta- 4r\theta - 2\theta^2 + 2r\theta^2) + a_3\theta + a_4(4\theta - 4\theta^2 - 8r\theta + 8r\theta^2) + 2a_5r\theta^2] + (r - s(\theta, t))^2 [a_1(-1 + \theta) + a_2(-2 + 2r + 4\theta - 4r\theta - 2\theta^2 + 2r\theta^2) + a_3\theta + a_4(4\theta - 4\theta^2 - 8r\theta + 8r\theta^2) + 2a_5r\theta^2] + (r - a)(r - s(\theta, t))^2 [a_2(2 - 4\theta + 2\theta^2) + a_4(-8\theta + 8\theta^2) + 2a_5\theta^2]$$

Also, the derivatives of w with respect to θ are:

$$\frac{\partial w}{\partial \theta} = -2(r-a)(r-s(\theta,t))\frac{\partial s(\theta,t)}{\partial \theta}[a_1(1-r-\theta+r\theta)+a_2(1-2r-2\theta+r^2+\theta^2+4r\theta-2r^2\theta-2r\theta^2+r^2\theta^2)+a_3r\theta+a_4(4r\theta-4r\theta^2-4r^2\theta+4r^2\theta^2)+a_5r^2\theta^2]+(r-a)(r-s(\theta,t))^2[a_1(-1+r)+a_2(-2+2\theta+4r-2r^2-4r\theta+2r^2\theta)+a_3r+a_4(4r-8r\theta-4r^2+8r^2\theta)+2a_5r^2\theta]$$

and

$$\frac{\partial^2 w}{\partial \theta^2} = -2(r-a)(r-s(\theta,t))\frac{\partial^2 s(\theta,t)}{\partial \theta^2}[a_1(1-r-\theta+r\theta)+a_2(1-2r-2\theta+r^2+\theta^2+4r\theta-2r^2\theta-2r\theta^2+r^2\theta^2)+a_3r\theta+a_4(4r\theta-4r\theta^2-4r^2\theta+4r^2\theta^2)+a_5r^2\theta^2]+2(r-a)\left(\frac{\partial s(\theta,t)}{\partial \theta}\right)^2[a_1(1-r-\theta+r\theta)+a_2(1-2r-2\theta+r^2+\theta^2+4r\theta-2r^2\theta-2r\theta^2+r^2\theta^2)+a_3r\theta+a_4(4r\theta-4r\theta^2-4r^2\theta+4r^2\theta^2)+a_5r^2\theta^2]-2(r-a)(r-s(\theta,t))\frac{\partial s(\theta,t)}{\partial \theta}[a_1(-1+r)+a_2(-2+2\theta+4r-2r^2-4r\theta+2r^2\theta)+a_3r+a_4(4r-8r\theta-4r^2+8r^2\theta)+2a_5r^2\theta]-2(r-a)(r-s)\left(\frac{\partial s}{\partial \theta}\right)[(-1+r)a1+a_2(-2+2\theta+4r-2r^2-4r+2r^2\theta+a_3r+a_4(4r-8r\theta-4r^2++8r^2\theta)+a_5(2r^2\theta)+(r-a)(r-s(\theta,t))^2[a_2(2-4r+2r^2)+a_4(-8r+8r^2)+2a_5r^2]$$

In addition, for the function ψ(r, θ, t) which satisfies ψ = 1 on the anode, in which ψ is given by:

$$\psi(r,\theta,t) = \frac{r-a}{s(\theta,t)-a} + \frac{(r-a)(r-s(\theta,t))}{s(\theta,t)-a}\left(\frac{\dot{s}(\theta,t)}{1+\left(\frac{\bar{s}(\theta,t)}{s(\theta,t)}\right)^2} - \frac{1}{s(\theta,t)-a}\right)$$

where $\dot{s}(\theta,t) = \frac{\partial s(\theta,t)}{\partial t}$ and $\bar{s}(\theta,t) = \frac{\partial s(\theta,t)}{\partial \theta}$

Now:

$$\frac{\partial \psi}{\partial r} = \frac{1}{s(\theta,t)-a} + \left[\frac{(r-a)+(r-s(\theta,t))}{s(\theta,t)-a}\right]\left[\frac{\dot{s}(\theta,t)}{1+\left(\frac{\bar{s}(\theta,t)}{s(\theta,t)}\right)^2} - \frac{1}{s(\theta,t)-a}\right]$$

$$\frac{\partial^2 \psi}{\partial r^2} = \left[\frac{2}{s(\theta,t)-a}\right]\left[\frac{\dot{s}(\theta,t)}{1+\left(\frac{\bar{s}(\theta,t)}{s(\theta,t)}\right)^2} - \frac{1}{s(\theta,t)-a}\right]$$

$$\frac{\partial\psi}{\partial\theta}=\frac{-(r-a)\bar{\bar{s}}}{(s-a)^2}+\left(\frac{(r-a)(r-s)}{s-a}\right)\left[\frac{\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]\dot{\bar{s}}-2\left(\frac{\bar{s}}{s}\right)\dot{s}\,\frac{s\bar{\bar{s}}-(\bar{s})^2}{s^2}}{\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]^2}+\frac{\bar{s}}{(s-a)^2}\right]+$$

$$\left[\frac{\dot{s}}{1+\left(\frac{\bar{s}}{s}\right)^2}-\frac{1}{s-a}\right]\left[\frac{(r-a)\left[(s-a)(-\bar{s})-(r-s)\bar{s}\right]}{(s-a)^2}\right]$$

and:

$$\frac{\partial^2\psi}{\partial\theta^2}=(r-a)\left[\frac{(s-a)\bar{\bar{s}}-2\bar{s}^2}{(s-a)^3}\right]+\left(\frac{(r-a)-(r-s)}{s-a}\right)\frac{\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]^2\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]\dot{\bar{\bar{s}}}}{\left[\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]^2\right]^2}+$$

$$\frac{\dot{\bar{s}}\left[2\left(\frac{\bar{s}}{s}\right)\frac{s\bar{\bar{s}}-\bar{s}^2}{s^2}\right]}{\left[\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]^2\right]^2}-\frac{\left(\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]\dot{\bar{s}}\right)\left(4\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]\left(\frac{\bar{s}}{s}\right)\frac{s\bar{\bar{s}}-\bar{s}^2}{s^2}\right)}{\left[\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]^2\right]^2}-$$

$$\frac{\dfrac{\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]^2(-2)\left[\frac{\bar{s}.\dot{s}}{s}\,\frac{s^2(s\bar{\bar{\bar{s}}}+\bar{\bar{s}}\ \bar{s})-(s\bar{\bar{s}})2(s\bar{s})}{s^4}\right.}{s^4}}{\left[\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]^2\right]^2}$$

$$\frac{\dfrac{-s^2(2\bar{s})\bar{\bar{s}}-\bar{s}^2(2s)\bar{s}}{s^4}+\left(\dfrac{s\bar{\bar{s}}-\bar{s}^2}{s^2}\right)\dfrac{s\left(\bar{s}\dot{\bar{s}}+\dot{s}\bar{\bar{s}}\right)}{s^2}-}{\left[\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]^2\right]^2}$$

$$\frac{\left.\frac{-(\bar{s}\dot{s}\bar{s})}{s^2}\right]-2\left[\left(\frac{\bar{s}\dot{s}}{s}\right)\left(\frac{s\bar{\bar{s}}-\bar{s}^2}{s^2}\right)\right]\left(2\left[1+\left(\frac{\bar{s}}{s}\right)^2(2)\left(\frac{\bar{s}}{s}\right)\left(\frac{s\bar{\bar{s}}-\bar{s}^2}{s^2}\right)\right]\right)}{\left[\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]^2\right]^2}+\frac{(s-a)^2\bar{\bar{s}}-\bar{s}^2(2)s}{\left((s-a)^2\right)^2}+$$

$$\frac{\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]\bar{\dot{s}}-2\left[\left(\frac{\bar{s}}{s}\dot{s}\right)\left(\frac{s\bar{\bar{s}}-\bar{s}^2}{s^2}\right)\right]}{\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]^2}+$$

$$\frac{\bar{s}}{(s-a)^2}(r-a)\frac{(s-a)(-\bar{s})-(r-s)\bar{s}}{(s-a)^2}\left(\frac{\dot{s}}{1+\left(\frac{\bar{s}}{s}\right)^2}-\frac{1}{s-a}\right)$$

$$\frac{(r-a)\left[(s-a)^2\left((s-a)(-\bar{\bar{s}})-\bar{s}^2-\right.\right.}{\left[(s-a)^2\right]^2}\ \frac{(s-a)\bar{s}(2(s-a)\bar{s})}{\left[(s-a)^2\right]^2}-$$

$$\frac{\left.\left.(s-a)^2\left((r-s)\bar{\bar{s}}-\bar{s}^2-(r-s)\bar{s}(2(s-a)\bar{s})\right)\right]}{\left[(s-a)^2\right]^2}+\ \frac{(r-a)\left[(s-a)(-\bar{s})-(r-s)(\bar{s})\right]}{(s-a)^2}$$

$$\frac{\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]\bar{\dot{s}}-\dot{s}\frac{2\bar{s}}{s}\frac{s\bar{\bar{s}}-\bar{s}^2}{s^2}}{\left[1+\left(\frac{\bar{s}}{s}\right)^2\right]^2}+\frac{\bar{s}}{(s-a)^2}$$

From the chemical and physical interpretation of the problem, and for the approximate solution, we propose the moving boundary $s(\theta, t)$ of this problem requires that the conditions must be satisfied:

1. When θ increases, then $s(\theta, t)$ increases.
2. When t increases, then $s(\theta, t)$ decreases.
3. When $t = 0$, then $s(\theta, t) = s_0(\theta)$ is the initial moving boundary.

The following definition of $s(\theta, t)$ may be used, which satisfies the above three conditions:

$$s(\theta, t) = s_0(\theta) - (a_6 + a_7(\pi - \theta)^2 t\, e^{a_8 t}, \quad 0 \le \theta \le \pi$$

and hence:

$$\bar{s}(\theta,t)=\frac{\partial s(\theta,t)}{\partial\theta},\quad \bar{\bar{s}}(\theta,t)=\frac{\partial^2 s(\theta,t)}{\partial^2\theta},$$

$$\bar{\bar{\bar{s}}}(\theta,t)=\frac{\partial^3 s(\theta,t)}{\partial^3\theta},\quad \dot{s}(\theta,t)=\frac{\partial s(\theta,t)}{\partial t},$$

$$\bar{\dot{s}}(\theta,t)=\frac{\partial \dot{s}(\theta,t)}{\partial\theta},\quad \bar{\bar{\dot{s}}}(\theta,t)=\frac{\partial^2 \dot{s}(\theta,t)}{\partial^2\theta}$$

where $s_0(\theta) = 1$, and the initial moving boundary:

$s(\theta, t) = s_0(\theta) - (a_1 t e^{a_3 t} + a_2 t e^{a_3 t} (\pi - \theta)^2)$

$\bar{s}(\theta, t) = -((a_2 t e^{a_3 t})(2(\pi - \theta))(-1)$

$= 2a_2 t e^{a_3 t} (\pi - \theta)$ .... (15)

$\bar{\bar{s}}(\theta, t) = -2a_2 t e^{a_3 t}$ .... (16)

$\dot{s}(\theta, t) = -a_1(a_3 t e^{a_3 t} + e^{a_3 t}) + a_2((a_3 t e^{a_3 t} + e^{a_3 t})(\pi - \theta)^2)$ (17)

$\bar{\dot{s}}(\theta, t) = -2a_2((a_3 t e^{a_3 t} + e^{a_3 t})(\pi - \theta))$...... (18)

$\bar{\bar{\dot{s}}}(\theta, t) = 2a_2(a_3 t e^{a_3 t} + e^{a_3 t})$...................... (19)

Now, substitute the derivatives for ψ_{rr}, ψ_r and $\psi_{\theta\theta}$, we get an equation with eight constants to be determined, namely $a_1, a_2, \ldots, a_8$. Therefore, we need for eight points in the region of definition say:

(0, 0), (π, 0), (2π, 0), (0, 0.5), (π, 0.5), (2π, 0.5), (0, 1) and (π, 1)

Hence, we get the a linear system of algebraic equations Aa = B, and upon solving this linear system using Mathcad computer softer, we get the following results:

$a_1 = 1.25\times10^{-3}$, $a_2 = 2.54233$,

$a_3 = 1.45\times10^{-4}$, $a_4 = 3.456\times10^{-4}$,

$a_5 = 1.43$, $a_6 = 0.0251$, $a_7 = 1.343$,

$a_8 = 0.6453$

Also, the graph of the moving boundary is given in Fig.3 and the solution of the ECM problem at time t = 0.1 is given in Fig.4:

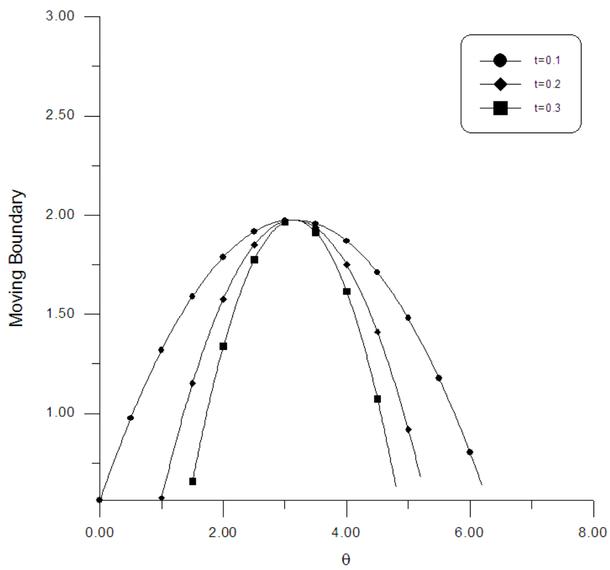

Fig.3: The moving boundary for different time step.

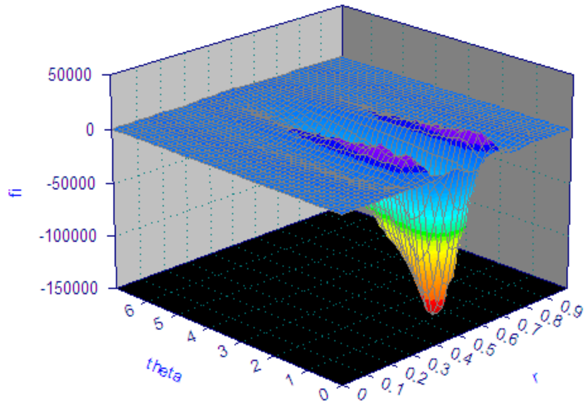

Fig.4: Solution of the ECM problem at time t = 0.1.

CONCLUSION

From Fig.3, one can see the accuracy and validity of the obtained results, since the anode is reduced or decreased with increasing time.

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الخلاصة:

في هذا البحث قمنا بحل المسئلة المكننة الكتروكيميائيا" باستخدام طريقة التجميع بالاعتماد على متعددة حدود برنشتاين .

هذا البحث ينقسم الى ثلاثة اقسام في القسم الاول قدمنا نظرة عامة مبسطة على المسائل المكننة الكتروكيميائيا" ذات البعدين بأستخدام الاحداثيات القطبية . وفي القسم الثاني تكلمنا عن متعددة الحدود برنشتاين متناولين فيها اهم التعاريف والخصائص لها واخيرا" في القسم الثالث قمنا بستخراج الحلول التقريبية للمسئلة واستخراج النتائج التي ساهمت في تحديد الاشكال المرفقة في نهايةالبحث التي توضح الحلول .